Synopsis

"Exploration of Accelerating Cosmological Models in Modified Theories of Gravitation"

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1. Introduction:

Gravity is the one of the most exiting thing attracted of the human being from thousands of years Gravitation, is a natural phenomenon by which all things with mass or energy including planets, stars, galaxies, and even light are attracted to one another. On Earth, gravity gives authoright to physical objects, and the Moon's gravity causes the ocean tides. The gravitational attraction of the original gaseous matter present in the Universe caused it to begin coalescing and forming stars and caused the stars to group together into galaxies, so gravity is responsible for many of the large-scale structures in the Universe. Gravity has an infinite range.

In 1686, Sir Isaac Newton was proposed the theory of gravitation. He gifts to the world the inverse square law. A great mathematician Albert Einstein in the year 1905, he proposed a special relativity in his paper, "On the electrodynamics of moving bodies". It is the physical theory of measurement in an inertial frame of reference. The Einstein's theory has been applied only for the special case of inertial reference frames i.e., frame of reference in uniform relative motion with respect to each other but the absolute motion of body cannot be measured by the principles of Einstein's special relativity with the help of the special relativity, author measure only the relative motion of body, hence special relativity does not help to gravitation in its structure. Therefore, special relativity fails to accommodate the fundamental ideas of relativity of all kinds of motion and gravitation. In this connection, Albert Einstein generalized the special relativity and Universal law gravitation, providing unified description of gravity as a geometric property of space-time. The general theory of relativity is simplest theory that consistent with experimental evidences. It is more accurate theory to describe the phenomenon of gravitation in the modern physics. General Relativity is the first among the mathematical Physics theory, leading the way for other mathematical theories in Physics such as the Gauge theories and String theories. General Relativity is based on following principles.

1. The principle of equivalence
2. The principle of covariance
3. Mach's principle

The general field equations i.e., Einstein's field equations which are associated with gravitational field are represented by

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -8\pi G T_{ij},$$

where R_{ij} is the Ricci tensor, R is Ricci scalar, T_{ij} is the energy momentum tensor, Λ is the cosmological constant and G is the Newton's constant of gravitation.

Alternative theories of gravity are physical theories that describe the concept of gravitation more satisfactorily for some physical aspects. The most important theories like Bergmann and Wagoner (1968,70a), Callan et al. (1970), Nordtvedt (1970), Hoyle and Narlikar et al (1963, 1966, 1971), Sen-Dunn et al (1971), Dunn (1974), Canuto (1977), Schmidt (1981), Barber (1982), Saez Ballester (1985) etc. have attempted to generalize the general theory of relativity. Scalar tensor theories of gravitation are considered to be important to explain the gravitational interactions near the Planck Scale: String theory, extended inflations and many higher order theories implying scalar field.

Resent observations shows that the present universe is accelerating and expanding. Despite the successes of General Relativity, there exist other theories of gravity known as modified theories of gravitation which are currently compatible with these observable data. Modified theories of gravitation are attracting more and more attention of researchers. This is because of the fact that these theories may serve as the possible candidate for explaining accelerated expansion. $f(R)$, $f(T)$, $f(R,T)$, f(G) and $f(R, G)$ are some of the modified theories of gravitation. $f(R)$ gravity theory was first proposed by Hans Adolph Buchdahl (1970). $f(R)$ theory of gravity is basically an extension of the usual Einstein-Hilbert action in gravitational theory of relativity with an arbitrary function of the Ricci Scalar R (Faraoni 2008). These models consist of higher order curvature invariants as different functions of the Ricci scalar. $f(R)$ gravity theory is a powerful approach to describe dynamical behaviour of the Universe. Recently, at a cosmological, galaxies and solar system scales interesting results have been achieved in the framework of the $f(R)$ gravity.

2. Review of Literature

In the last decades the several theories of gravitation have been proposed as an alternative to Einstein's general theory of relativity.

Brans and Dicke (1961) have formulated a scalar tensor theory of gravitation in which the tensor field alone is geometrilized and scalar field is alien to the geometry.

Sen and Dunn (1971) have shown a specially homogeneous and anisotropic cosmological model in a scalar tensor theory. This theory is obtained when the source of gravitational field is a perfect fluid with pressure equal to energy density and the metric is of the Bianchi type I. Various physical properties of the model have also been discussed

Chandel and Shri Ram(2014) investigated that the study of particle creation and bulk viscosity in the evolution of specially homogeneous and anisotropic Bianchi type V. Cosmological model in the framework of Saez- Ballester theory of gravitation.

Kanakavalli et al. (2016) investigated the Bianchi type III cosmological model in the frame of work of *f(R,T)* gravity in the presence of two non interacting matter sources .They have solved the field equation using the fact that scalar expansion is proportional to shear scalar of the space time and presented the scalar field model in this theory. Finally they have also discussed some physical and kinematical properties of the model.

Riess et.al. (1998) and Perlmutter et.al.(1999) has investigated accelerated expansion of the universe has revolutionized the modern cosmology . This acceleration is realized with negative pressure and positive energy density that violate the strong energy condition. This violation gives a reverse gravitational effect due to this effect, the Universe gets a jerk and the transition from the earlier deceleration phase to the recent acceleration phase takes place . The cause of this sudden transition and the source of accelerated expansion are still unknown. The state of the art in cosmology has led to the following present distribution of the energy densities of the Universe: 4% for baryonic matter, 23% for non-baryonic dark matter and 73% so-called Dark Energy . The exact nature of dark energy is not clear even today. In that case there are several dark energy candidates. One of the candidate of the dark energy is Einstein's cosmological constant which has two crucial problems so called 'fine tuning' and 'cosmic coincidence' . There are also other models to describe dark energy such as k-essence model and tachyon model. An interesting model to describe dark energy is Chaplygin Gas . Kamenshchik et al (2001) that are based on Chaplygin equation of state (Chaplygin et al 1904) which are not of good consistent with the observational data . On the other hand Chaplygin Gas is an interesting fluid explaining the acceleration of the universe. Further extension of Chaplygin Gas model called Modified Cosmic Chaplygin Gas. Chaplygin Gas model were studied from the holographic point of view .

Yadav and Sharma (2013) present a model of transitioning universe with minimal interaction between perfect fluid and anisotropic dark energy in Bianchi I space-time.Reddy et.al.(2012) have studied five dimensional Kulz-Klein model in f (R,T) theory of gravitation.They observed that the model has no singularity and quite similar to the case of five dimensional f (R) gravity. Sahoo et. al. (2014) have studied Kulz-Klein metric for a perfect fluid distribution in the framework of modified theory of gravitation. They have shown that the model reduce to the usual relativistic equation. Rao et al.(2015) have investigated five

dimensional Friedmann Robertson Walker metric with holographic dark energy in the frame work of Brans and Dicke scalar–tensor theory of gravitation. To obtain a determinate solution of the field equations they have used a particular relation between scalar field and the scale factor $a(t)$ of the universe. They have discovered Universe stands as a major breakthrough of the observational cosmology

Pawar et al (2008) investigated that tilted of cosmological models in $f(R,T)$ theory of gravitation without taking any relation between density ρ and pressure p. Mohanty and Mahanta (2008) have shown that homogeneous plane symmetric string cosmological model to Takabayasi string in Barberself creation theory.

In 1982, Barber proposed two theories of gravitation . These theories are known as self creation theories. First theory is modification of Brans Dicke theory and second theory is modification of ganaral relativity. However the first self creation theory violates equivalence principle. Pawar et al (2009) studied titled plane Symmetric cosmological model in presence of perfect fluid with heat conduction.

Khadekar and Patki (2005) studied Kaluza - Klein universe with dynamical cosmological term. Singh and Sharma(2010) analysed Bianchi type II cosmological model in Brans –Dicke theory. They have examined the energy conditions .Varma and Ram (2010) studied Bianchi type III space time with bulk viscosity and time dependent G and $\wedge$ .They have shown that the universe is expanding shearing and non rotating. Bali and Chandanani (2009) investigated Bianchi type III space time in presence of perfect fluid with time dependent displacement vector for Barotropic fluid distribution in Lyta geometry.

3. Aim:

Our aim is to study cosmological model for explaining the acceleration expansion of the universe. We proposed to the study of various forms of space time in the theory of gravitation. The present research may lead to explor some hidden aspects of the universe.

4. Objective:

1. To extend the present work.
2. Analysis some cosmological model in presence of different cosmic fluid.
3. To investigate the physical, geometrical, kinematical properties and applications of existing models.
4. To investigate cosmological model in presence of various fluid such as dark energy ,string, domain wall, holographic dark energy , viscosity e.t.c
5. To ascertain the reliability, consistency and compatibility of the theories under investigation as regard to their predictions in view of observational data.
6. To study FRW , Bianchi type and higher dimensional cosmological models in theory of gravitation.

5. Methodology

For the proposed entitled work, we will collect the relevant documentations such as research papers, reference books from various resources like reputed e-libraries, NISCAIR, New Delhi, University Library etc. We will analyse the research papers which are published in reputed journals relevant to the research proposed. Further, We will do Mathematical computational work for solving the field equations employing the existing methods of solving partial differential equations. In case of difficulties the numerical method may be adopted. We will construct and present the research papers in workshops/conferences/seminars for up gradation of quality of our research papers.

6. Important of Study

From ancient times the universe has become matter of curiosity and wonder to the human kind. There are so many interesting things in the universe. The main purpose of this work is to examine and explore the hidden aspects of the universe. Our investigation from a piece of academic treasure of documentation emerging as new facts with relevant interpretation which may lead to new cosmological models which will help to decide the importance of gravitational theories.

7. Plan of Research Work:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic.	Good command on the subject and clear the concepts.
2	Second year	Collecting the papers and analyze them.	To understand the mathematical techniques of Research Paper
3	Third Year	Work on some problems to be included in the thesis.	Submit at least two papers.
4	Fourth Year	Publishing some papers and submitting the Thesis.	Some more papers and Thesis submission.

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